OPTIMAL MONETARY POLICY WITH OUTPUT AND ASSET PRICE VOLATILITY IN AN OPEN ECONOMY: EVIDENCE FROM KENYA

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Abstract

This paper attempts to establish optimal response of monetary policy to output, inflation and asset price volatility in a small open economy, taking into account optimisation behaviour of households and firms. The empirical estimates suggest that monetary policy responds to deviation of interest rate and output growth rate from their targets with greater weight compared to asset prices and inflation. The dynamic optimisation results are consistent with empirical estimate of weights monetary policy uses when stabilising output and inflation. Furthermore, numerical solution of dynamic optimisation model indicate that commitment to a monetary policy rule achieves output and inflation objectives as well as higher welfare compared to discretionary monetary policy rule.

1 Introduction

The intention of monetary authority in most market economies is to maintain price stability, so that price mechanism can allocate an economy's resources efficiently and in a way that is socially desirable. An efficient as well as a socially desirable resource allocation leads to distribution of income and wealth that is acceptable from the society's point view. In addition, it not only accelerates the rate of economic growth, but also stabilises output growth rate. High and stable output growth rate encourages investment and increases utilisation of resources like labour, which improves income of households as well as welfare, especially in developing countries where living standards are lower than in developed countries. Therefore, monetary policy, by achieving price stability, also achieves the objective of output growth rate stability without any prejudice to inflation stability. In particular, effectiveness of monetary policy of inflation stabilisation depends on its capacity to influence market interest rate and expectations of private agents about future interest rates and prices (Woodford, 2003, 2013). From the monetary policy transmission mechanism, a change in monetary policy interest rate to stabilise inflation affects liquidity and market interest rates. The market interest rates allocate credit and, as such, influence investment and consumption in the economy. In this regard, inflation stabilisation objective can be achieved by managing investment and consumption. Furthermore, in a demand driven economy, changes in consumption and investment as a results of interest rate adjustment can drive the level of actual output towards potential or target output.

Another way in which inflation can be stabilised by monetary policy is by controlling inflation expectations. Forward looking agents, based on available information, make expectations about prices which influence their current consumption and investment decisions. Monetary policy is one of the basis upon which forward looking agents form expectations about the evolution of future commodity and asset prices (Kuttner, 2001; Bernanke & Kuttner, 2005). Hence, monetary policy actions that are contrary to the expectation induce uncertainty. This makes it difficult for agent to form expectation about future prices. The uncertainty in monetary policy also undermines prediction of liquidity, financial and commodity prices by households and firms. As a result, households and firms change their consumption and investment in a bid to insure themselves against risk associated with liquidity and price uncertainty, leading to a substantial deviation of output and prices from their targets. Conversely, a consistent and credible monetary policy actions informs expectations in the private sector, thereby reducing uncertainty with respect to interest rates and asset prices. Consequently, the resultant financial asset prices are not only stable, but also reflect the discounted present value of asset earnings.

Stable asset prices safeguard the stability and integrity of the financial market. This enables it to reallocate excess investible funds from savers to investors in need for funds to invest. It also increases stability of financial market because financial asset prices that deviate from discounted present value of asset distort the balance sheet of households and firms. As a result, on the one hand, households and firms do not optimise their debt holding. This leads to either over accumulation of debt that drives agents into liquidity crisis and financial distress, since lenders are unwilling to lend firms and households in financial distress. Or, households and firms, even though they may have a shortage of investible funds, may borrow too little to be sufficient for capital formation required to achieve the target growth rate.

On the other hand, investors in the financial markets with information asymmetry[1] rely on the balance sheets to assess the credit worthiness and the viability of their potential investment. Hence, inaccurate balance sheets provide wrong information about networthy of the investment that predisposes investors to losses, which reduces investment. Hence, an economy wide distortions in the balance sheets due to asset price misalignment affects investment as well as deviation of output from potential output. Monetary policy intervention is thus required to align asset prices to their fundamental value, to stabilise output and thereby, achieve output stabilisation objective (Bernanke & Gertler, 2000; Mishkin, 2001). Therefore, monetary policy that achieves output and inflation targets as well as financial asset prices that are close or the same as their discounted present value of asset earning, is optimal and maximises welfare.

However, given that financial asset price misalignment occur during and after periods of stable inflation, a monetary policy response to financial asset price instability may require a change in interest rate that is inconsistent with actual and expected inflation (Schwartz, 1995; Borio & Lowe, 2002). In this case, a response of monetary policy to stabilise financial asset prices causes

[1] Lenders do not have full information about the quality of the borrowers of investible funds and the viability of investment projects to be financed by the funds borrowed. Therefore, lenders require collateral or borrowers signal their quality by providing assets that can be sold to recover the funds borrowed should the borrower default (Stiglitz & Weiss, 1981)

inflation and output to deviate from their targets. The financial asset instability also reduces the ability of monetary authority committed to an inflation target to credibly signal that it will achieve the target. This increases uncertainty in monetary policy. Therefore, asset price instability compromises the effectiveness of monetary policy to address deviation of inflation and output from their targets, which undermines stability of monetary policy (Bernanke & Gertler, 2000; Mishkin, 2001).

In addition, an economy with supply side constraints and economic agents with substantial market power that enables them to influence prices, reduce sensitivity of prices to monetary policy stimuli. As a result, monetary policy action intended to stabilise prices leads to either a deviation of output from potential output, or a deviation of both price and output from their targets. In as much as monetary authority emphasises price stability as one of the main objective of monetary policy, existence of shocks in the real and financial sector as well as the changes in exchange rate due to shocks in the foreign sector influence output and price stability especially in small open economies. The fluctuations in commodity exports, imports and also international portfolio investment flows affect the quantity of foreign reserves in the economy, which influence nominal exchange rate. Movements in the nominal exchange rate have a direct effect on aggregate demand, as they alter the relative prices of foreign and domestic good as well as consumer price index in so far as they affect cost of inputs and imported consumer goods.

In addition, movement in exchange rate affect actual and expected yield on financial assets, which influence asset price volatility in the financial market. Therefore, monetary authority is compelled to change interest rate to address instability in inflation and financial asset prices in the domestic economy caused by exchange rate movements. Yet, the resultant interest rate differential between the domestic and the foreign financial markets cause portfolio flows that again trigger exchange rate movement. Furthermore, with output and inflation stability targets, interest rate changes, for instance, to accelerate output growth rate to achieve target growth rate requires low interest rate, but this causes nominal exchange rate depreciation and inflation. This generates a trade-off in an economy with a strong exchange rate pass-through in achieving output growth rate and inflation targets.

Therefore, the main objective of this paper is to analyse optimal response of monetary policy to output, inflation and asset price volatility in order to stabilise asset price, output and inflation in Kenya. To this end, the paper develops a social welfare function, from the general equilibrium model. Then the social welfare function is solved numerically using the dynamic programming method of Bellman & Lee (1984), taking into account optimisation decisions of households and firms. An approach that has been emphasized and used by Rotemberg & Woodford (1997), Benigno (2004) and Divino (2009) in their analysis of optimal monetary policy. The formulation and numerical solution of the social welfare problem as dynamic program enables estimation of the time path of monetary policy rate that maximises social welfare. The weights estimated indicate optimal response of monetary policy to deviations of objective variables from their target, considering the static and dynamic responses of agents to monetary policy actions in an economy. This enables the evaluation of monetary policy outcome under discretion and consistency rules.

The parameter estimates of the social welfare function indicate that monetary authority should respond with greater weight to deviation of output from the target compared to deviation of asset prices and inflation from their respective targets in order to maximise welfare. With respect to optimal rules, dynamic programming results indicate that commitment to a monetary policy rule leads to higher welfare than discretion. The findings are corroborated by regression estimates of the quadratic loss function. The results suggest that, as much as output, asset price and inflation stabilisation are important; output stability has the greatest influence on welfare and hence should be prioritised among the monetary policy stabilisation objectives. The finding are consistent with Woodford (2003) and Divino (2009) analyses of optimal monetary policy.

There is dearth of literature on the optimal response of monetary policy to output and inflation stability, for example (Clarida et al., 2001; Woodford, 2003; Benigno, 2004). However, few studies investigate optimal response of monetary policy when asset prices, inflation and output stability are the main objectives of monetary policy. Yet, instability in asset prices, besides affecting the financial sector and monetary stability, reduce investment and exert recessionary effect on the economy. The interest rate changes as a result of monetary policy actions to address asset price and output fluctuations cause exchange rate movements, which affects inflation. This paper attempts to fill this gap by analysing the optimal response of monetary policy on output and asset price stability, considering price stabilisation objective. This approach provides a more comprehensive analysis of the monetary policy stabilisation problem faced by monetary authorities in small open developing economies. This paper is related to Rotemberg & Woodford (1997), Benigno (2004) and Divino (2009) with respect to the analysis of optimal response of monetary policy to output, interest rate and exchange rate instability respectively.

The main innovations in this paper are, first, it includes equity and bond prices in establishing weights with which monetary authority should respond to deviation of variables of interest from their desired level. This approach allows the analysis of differences in monetary policy response to equity and bond price instability given that they respond differently to changes in interest rates. Secondly, the social welfare criterion for evaluating optimal monetary response is solved numerically using dynamic programming method of Bellman & Lee (1984) to estimate optimal penalty imposed on deviation of variables of interest to monetary authority from the target and the optimal time path of policy rate. In this way, the policy rate is the decision variable, whose path is used to evaluate the effect of consistent and discretionary monetary policy rules for inflation, output and asset price gaps as well as welfare. The effectiveness of monetary policy depends on how a change in monetary policy instrument affects market interest rate and also the signal it sends to the market, to inform rational expectations about market interest rates. This is important because, in a market economy, interest rates and financial asset prices allocate credit and capital. Therefore, monetary policy actions that influence evolution of interest rates overtime to the desired path leads to a higher welfare, if the interest rates expectation of the agents coincides with those of monetary authorities (Kydland & Prescott, 1977). Such a monetary policy regime achieves monetary policy objectives. This is essential for enhancing much needed investment and economic growth in developing economies like that of Kenya.

The rest of the paper is organised as follows. The next section explains optimal monetary policy intervention in the economy. Sections 3 and 4 discusses the methodology, data and main results respectively, while section 5 summarises issues from the main results. The section concludes that a consistent monetary policy rule not only achieves price and output stability objectives, but also attains a higher welfare compared to a discretionary monetary policy rule.

2 Optimal Monetary policy stabilization of Output and Prices

The views on optimal monetary policy start from the perspective that an economy can achieve an efficient level of inflation, output and employment, if there are no distortions in price like rigidity in wages and prices. The corresponding level of inflation and output maximise individual and social welfare. Such an outcome cannot be improved upon without prejudice to welfare of other agents in the society, and hence a monetary intervention only leads to inefficient outcome with a lower level of welfare. This is the so called "best equilibrium". The implication of the best equilibrium outcome is that, monetary policy price intervention can lead to prices that distort incentives to work, save and invest. As a result, output growth rate may not only be too high or too low to maximise welfare, but also cause instabilities in the growth rates of output and prices[2]. Therefore, so long as the equilibrium results from economic agents facing the right prices, a monetary policy intervention in the economy is not required (Rotemberg & Woodford, 1997; Svensson, 1997).

However, distortions in the economy caused by shocks, market power of agents on the demand, and supply side and information asymmetry among agents during transactions establish distorted price. Price distortions either misallocate or lead to under utilisation of an economy's resources. This results to actual output and inflation deviating from potential output and inflation target respectively. Even if there are no price distortions, such that established prices lead to efficient utilisation of resources and hence efficient output level, the resultant inflation and output can be undesirable from the point of view of the society (Svensson, 1997; Benigno, 2004). Consequently, monetary policy intervention is required to remove the price distortions, which leads to the so called "first best equilibrium". In particular, monetary authority can alter prices by either changing liquidity or policy interest rate, to influence agents' behaviour, so that agents can adjust their saving, investment and consumption as well as expectation such that a socially desirable inflation and output level is attained. This is relevant for market economies in the sense that, current and future commodity and financial asset price distortions exist in the economies. For instance, labour unions, market power, changes in demand for goods, services and capital as well as natural factors, influence domestic prices. In addition, developing economies have levels of national outputs that

[2] This is the first fundamental welfare theorem, which presupposes that in an economy without distortions, prices allocate resources in such a way that any subsequent reallocation results to inefficiencies that reduces welfare of some agent. Such an allocation is pareto optimal. In this case there is no need of government intervention.

are lower than it is socially desirable. Hence, there is need of monetary policy intervention to accelerate output to potential output and to the level that is socially desirable.

Even though Svensson (1997) and Benigno (2004) imply that monetary policy intervention in prices and level of output is required to either remove distortion or achieve social welfare objective, they do not take into account the instability that may result from policy intervention. The instability emanates from the fact that monetary policy under the commitment rule, which yields second best equilibrium after removing distortion, does not take into account agents' expectations and optimisation decisions. In addition, the rules that guide intervention, may not incorporate new information that is relevant for intervention to improve outcome of a consistent monetary policy action (Kydland & Prescott, 1977). As a result, a consistent monetary policy yields welfare level that is suboptimal compared to the first equilibrium when a price distortion is removed (King, 1997; Kydland & Prescott, 1977; Svensson, 1997).

Nevertheless, discretionary policy overcomes weakness in commitment, by taking into account optimisation decisions of the agents, when intervening to address distortions each time. This affords monetary authority flexibility to respond to unanticipated price and output changes as well dynamic optimisation behavior of economic agents, thereby achieving an optimal policy intervention outcome. Indeed, this is relevant for the financial markets in which investors make decisions frequently to optimise their portfolio holding. In addition, small open developing economies like that of Kenya are susceptible to unpredictable capital flows and terms of trade changes, which affect asset and commodity prices respectively. Hence, there is need for monetary authority to apply discretionary monetary policy rule to address price stability which also affect output stability.

Yet, discretion results in, first, higher volatility in either prices or output. This is because, discretion does not provide agents with rules and information that they can use to make current decision to optimise. Consequently, as a result of inconsistency of a discretionary monetary policy intervention, agents adjust their decision as they get information, which affects stability of prices and output. Indeed, inconsistency in monetary policy action is a major cause of volatility in forward looking markets as it increases uncertainty (Svensson, 1997; McCallum, 1999; Woodford, 2003). Secondly, discretion allows monetary authority to use its superior information compared to the public, with respect to changes in the monetary policy instrument, to pursue its main objective with ease at the expense of other objectives perceived important by the public. This is the case where the public cannot observe and verify actual changes in liquidity caused by actions of the Central Bank. King (1997) argues that monetary policy under discretion does not consider inflation expectations of the public when responding to output shocks. As a result, stabilising output by changing liquidity increases instability in expected inflation as well as actual inflation. In this case, even though monetary policy manages to stabilise output, it does not effectively stabilise inflation. This result in discretion yielding a lower welfare compared to commitment.

In spite of the inconsistency of a discretionary monetary policy intervention that makes it fourth best equilibrium policy intervention[3], Rogoff (1985) asserts that discretionary intervention can be

[3]Government intervention in the market by removing distortion results in the first best equilibrium; intervention with a commitment

improved by appointing an agent whose social preferences are the same as those of the society. In particular, a more conservative agent with respect to output and inflation stabilisation, than the society employs his/her discretion to achieve price and output stability. However, the agent's conservative intervention may yield price and output level that is less desirable from the society's perspective. Rogoff (1985) and King (1997) establish that a conservative agent is inflation bias, when the society prefers more growth compared to price stability. However, the social preference as depicted by social welfare function, may require intermediate combination of growth and price stability, that is a combination of positive growth and non-zero average inflation rate. In particular, in Kenya, as it is in other developing countries where income growth objective is important, a positive inflation rate is required to encourage saving and investment by making nominal interest rate positive[4]. Hence, a conservative agent achieves an allocation that is inferior to an agent with commitment, but superior to pure discretion.

Nonetheless, the society can do better with an agent who has built a credible reputation of embodying social preference when intervening in the economy. An agent with such reputation acts with discretion to maximise individual welfare. Since, the preferences of the agent and public are the same, the outcome of the agent's intervention satisfies social preferences. Consequently, social welfare is maximised. This implies that, on the one hand, the agents' choice of price level and corresponding output are consistent with social expectation and preferences. On the other hand, a reputable agent provides credible information to the public, relevant for making current decisions. More importantly, the public has a rule to follow when making optimisation decision. Therefore, a reputable agent achieves output and price stability objective that is similar to commitment (Rogoff, 1985; Lockwood et al., 1998). One main weakness of reputation under discretionary monetary policy intervention is that the agent may not be effective in responding to unforeseen price distortions that the agent has no reputation of dealing with. For example, a Central Bank which has a reputation of maintaining price stability, cannot credibly address shocks that distabilise output, exchange rate and asset prices.

Empirical analysis of optimality of monetary policy intervention in the economy evaluates social welfare outcomes of intervention of monetary policy with commitment and discretion in general. The analyses focused on the approaches that achieve price stability, because when prices are stabilised at a level that enables a socially acceptable allocation and production, they lead to maximisation of social welfare. Therefore, optimality of monetary policy can be assessed with respect to the monetary policy rule that stabilise actual and expected inflation. For example Rogoff (1985), Rotemberg & Woodford (1997), Svensson (1997) and King (1997) analyse the effectiveness of commitment and discretion in maintaining inflation stability. The studies establish that commitment to an interest rate rule, when responding to high inflation, leads to inflation stability, although output growth rate deviates from the target. The interest rate rule also leads

rule leads to a second best allocation; discretion with a conservative agent achieves an allocation that is third best; while discretion leads to fourth best equilibrium (Barro & Gordon, 1983; King, 1997; Svensson, 1997). This intervention is consistent with second fundamental welfare theorem. Of course, public sector intervention in the economy may undermine efficiency with which a price mechanism allocates resources.

[4] The Central Bank of Kenya has a stated target range of $5 \pm 2.5\%$ for core inflation (CBK, 2015).

to higher instability in output despite inflation and interest rate being stable. Woodford (2003) focuses on a consistent interest rate adjustment by monetary authority in response to inflation changes. In this analysis, smooth adjustment of interest rate in the expected direction informs the expectation of the public, who optimise based on the information available. The study establishes that interest rate adjustment with inertia has a higher welfare compared to surprise adjustment. The inertial adjustment of interest rate provides information required for formation of expectation about future asset price, which reduce uncertainty in forward looking markets.

Divino (2009), Clarida et al. (2001), Corsetti & Pesenti (2005) and Benigno (2004) extend the analysis to include exchange rate. In this way, consider distortions in the domestic prices emanating from foreign economies. Their analytical solution of the social welfare function establishes that a consistent interest rate adjustment to stabilise inflation leads to output instability, because changes in interest rate, affects exchange rate, which then causes a deviation of output from the socially desirable level[5]. In this case, inflation stabilisation efforts by the monetary policy causes exchange rate distortions which reallocates resources, resulting in an equilibrium that is socially undesirable. There are a few limitations to these analyses. Firstly, though consistent with second best equilibrium, it does not take into account financial asset prices, like bond and equity prices when monetary policy is responding to price distortions. As long as equity and bond prices, which result from optimisation decision of agents on the financial market, are not taken into account, a discretionary intervention cannot do better than a commitment rule. Secondly, equity and bond prices are a source of information that can be used by monetary authority to stabilise inflation. Finally, in the general analytical solutions of Divino (2009), Clarida et al. (2001), Corsetti & Pesenti (2005) and Benigno (2004), social optimisation problem with monetary policy intervention do not yield numerical results that are easy to compare.

Therefore, this paper tries to overcome these weaknesses by analysing optimal monetary policy response to output and asset price volatility. A welfare analysis approach is used, because optimal intervention is more effectively evaluated by comparing welfare outcomes of alternative interventions. In addition, the welfare optimisation approach takes into account the optimisation decision of the public, which is relevant for monetary policy. The social problem is also solved using dynamic programming numerical method, which allows a quantitative comparison.

3 Methodology and Data

An optimal monetary policy is one that maximises social welfare, given the optimal decision of the public. The optimisation decision of the public involves a choice of consumption, money balances and leisure for given prices, interest rates and wages. Therefore, the price level that allows individuals to optimise their consumption, leisure and financial asset holding, leads to social welfare optimisation, as depicted by the social preferences. A good approximation of the

[5]For example, increase in inflation requires an increase in interest rate. The higher domestic interest relative to foreign interest rate attracts portfolio investment in the economy leading to an exchange rate appreciation, that reduces exports and increases import. This, drives output away from the potential output.

social welfare function is the second order Taylor approximation of the household utility function Rotemberg & Woodford (1997). (Divino, 2009) and Woodford (2003) use a similar approach. Changes in money balances held by the public caused by inflation and asset price movements as a result of monetary policy have a direct effect on the liquidity services money provides. Besides, liquidity services, inflation and asset prices affect the amount of wealth and disposable income and hence, consumption of goods and services. Therefore, the use of money in the utility function to approximate social loss function allows the effect of monetary policy on social welfare, as a result of changes in money balance and asset prices, to be captured. Even though a separable utility function with respect to consumption and leisure can be used as in Woodford (2003), money in the utility function with leisure is more appropriate, because it is broader. In this regard, a second order Taylor approximation around the steady state of the money in the utility function is used to approximate the social welfare function. This is the quadratic loss function given by equation 1 below.

$$L_t = \sum_{t=0}^{\infty} \beta^t \left(\gamma_\pi \pi_t^2 + \gamma_y y_t^2 + \gamma_Q \widetilde{Q_t}^2 + \gamma_B \widetilde{B_t}^2 \right) \tag{1}$$

$$y_t = \overset{*}{y} + x_t + b((1+i) - 1.2) + e_t \tag{2}$$

$$\pi_t = ((1+i) - \pi_{t+1}) + Cx_t \tag{3}$$

$$Q_t = \frac{(Q_{t+1}\pi_t\varepsilon_t)}{1+i} \tag{4}$$

$$B_t = \frac{B_{t-1}}{(1+i)} \tag{5}$$

Where L_t is the social loss to be minimised, γ_π, γ_y, γ_Q, γ_B are contributions of inflation, income, equity price and debt instability to social loss respectively. $\widetilde{B_t}$ is the debt level, y_t income, π_t inflation, $\widetilde{Q_t}$ equity price, $\overset{*}{y}$ potential income, β discount factor, ε_t nominal exchange rate. x_t is the public deficit, C contribution of public deficits to inflation and t is time. The details of derivation are provided for in appendix IV. Indeed equation 1 depicts preferences of the public with respect to inflation, output, equity and bond prices, which also affect consumption, leisure, and money balances. An optimal monetary policy action is the one that leads to price, interest rate, bond and equity prices, that minimise equation 1, subject to the first order condition of the utility maximisation problem of the public given by equations 2 to 5. Equation 2 is the Lucas aggregate supply function; 3 describes change in inflation overtime; equations 4 and 5 are the equity price and debt stock equations, respectively obtained from the household optimisation decision. This specification takes into account the fact that monetary policy cannot fully influence rational expectations of the public. This affects their optimisation decisions in relation to monetary policy actions. Following Lucas & Stokey (1983), monetary authority in this case choose a policy rate that enables an allocation of credit that maximises social welfare given the resource constraint and the decisions of agents in the economy.

In particular, a monetary authority chooses a target interest rate level and decides on the time path the policy interest rate should follow to stabilise inflation. The time path of the monetary policy rate then influences the time path of the market interest rate. Therefore, an optimal monetary policy intervention requires the entire path of monetary policy interest rate to minimise the social welfare function over the entire path. Thus, the solution a monetary authority provides to the social optimisation problem depends on the monetary policy rule it adopts, when responding to instability in the variables in the social welfare function.

A commitment policy rule entails solving optimisation problem once and for all, and the monetary authority applies the same optimal rule without deviation to maximise social welfare overtime. In particular, the monetary authority commits to an interest rate rule to minimise L_t. Thus, the problem can be recast as a dynamic optimisation problem, whereby L_t is a state variable and monetary policy interest rate is the decision or policy variable. Therefore, under commitment, the Central Bank commits credibly to a particular rule, when intervening in the market to stabilise price such that L_t is the lowest. However, under discretion, equation 1 is solved subject to the private sector's optimisation without commitment to a policy rule. This specification allows the application of dynamic programming method of Bellman & Lee (1984) to solve for the optimal interest rate path. The same approach has been used by Woodford (2003), though Söderlind (1999) argues that optimal response can be estimated using regression methods. However, regression estimates of the quadratic loss function parameters are average weights that do not reflect optimisation behaviour of the Central Bank and the public as well as reaction of agents, to policy decision (Rotemberg & Woodford, 1997). The next section presents estimates of the social welfare function weights. The estimates are based on quarterly data.

4 Main Results

Table 1 presents the least square estimates of the reduced form of quadratic loss function, with repo rate as the monetary policy rate[6]. The parameters on the loss function indicate the weight monetary authority attach to the variable when changing monetary policy to maximise the society's welfare. Thus, the coefficient of the estimated quadratic loss function is the penalty monetary policy imposes on the deviation of the variable in the social welfare function from the target or its long run level.

[6] log linearising and taking deviations from the steady of **??** which is the consumption Euler, and using the idea that $y_t = c_t$ gives $y_t = E_t y_{t+1} - \sigma\left[i_t - \varepsilon_t - E_t\pi_{t+1} - r_t + \delta\right]$ and using the Philips curve $\pi_t = \frac{(1-\Theta)(1-\Theta\beta)}{(\Theta)} mc_t + \beta E_t(\pi_{t+1})$ to substitute for $E_t\pi_{t+1}$ and making i_t the subject gives the equilibrium evolution of output y_t, inflation π_t, exchange rate ε_t, equity prices Q_t, and bond prices B. In this case, goals of stabilisation can be described in terms of output, inflation rate, interest rate and asset prices. The objective of monetary policy is to minimise aggregate deviations of output, inflation interest rate and asset prices each period, which is the quadratic loss function. This specification is similar to Woodford (2003) except for the inclusion of asset prices.

Table 1: **Quadratic Loss function**

	Repo Rate	**Repo Rate**	**Repo Rate**	**Repo Rate**	**Repo Rate**
	1	2	3	4	5
Output gap	-0.073		-0.037*	-0.017**	
	(0.100)		(0.011)	(0.008)	
Output growth rate		-0.078			-0.080*
		(0.162)			(0.011)
Exchange rate	0.320***	0.312*	0.156*	0.138*	0.121*
	(0.039)	(0.038)	(0.008)	(0.006)	(0.005)
Equity prices	-0.018	-0.012	-0.015*	-0.012*	-0.004*
	(0.011)	(0.009)	(0.001)	(0.001)	(0.001)
Inflation	0.019	0.019	0.009	0.006	0.028*
	(0.033)	(0.032)	(0.008)	(0.005)	(0.002)
Bond prices	0.031***	0.025	0.0402*	0.003	0.007***
	(0.017)	(0.016)	0.004)	(0.006)	(0.004)
interest rate	0.371*	0.369*		0.312*	0.356*
	(0.029)	(0.028)		(0.0186)	(0.021)
constant	1.329	1.420		2.080*	-0.068
	(1.206)	(1.165)		0.100	(0.130)
Volatility				0.005*	0.717*
				(0.001)	(0.078)
R-squared	0.662	0.655			
arch			2.205*	2.554*	2.383*
			(0.373)	(0.350)	(0.184)
garch			0.085*	0.003	0.122*
			(0.032)	(0.023)	(0.017)
F-statistic	53.10504	55.963			
Prob(F-statistic)	0.000	0.000			

Notes: In this regression the dependent variable is the deviation of repo rate from its trend. Output gap is the deviation of actual output from the potential output, which is the trend. Output growth rate is squared deviation from target growth rate. (Output is equal to the national income) inflation is the squared deviation from the average target inflation rate of 5 per cent. Equity, bond prices and interest are squared deviation from their long term trends. With respect to bond prices, the long term trend is consistent with the yield curve.

The regression estimates are valid given that endogeneity test in Table 2 in appendix IV indicates that there is no endogenous relationship between the dependent and independent variables in the model. In column 1 of table 1, the social welfare loss function is estimated with output gap. The first regression uses squared deviation of output from the potential level. The coefficient on output gap indicates that if output deviates from potential output by one unit increase, the policy rate reduces by 0.073 per cent, but the reduction in the policy rate is statistically insignificant. However, monetary authority responds to a 1 per cent deviation of exchange rate by imposing a penalty of 0.32 per cent. With respect to stock market, on the one hand, the Central Bank increases the policy rate by 0.031 per cent statistically significantly when bond price deviate by 1 per cent. On the other hand, the Central Bank does not respond to deviations of equity prices from the long term trend statistically significantly. This indicates that the Central Bank, in its effort to maximise social welfare, does not give significant weight to deviation of output and equity prices from their potential levels. A explanation is that developing economies have abundant resources like labour and natural resources that are grossly underutilised. As a result, they have a high potential output as well a large output gap. The monetary policy efforts to increase resource employment by increasing aggregate demand causes inflation. This is due to the presence of structural rigidities, that impede responsiveness of supply of goods and services and hence demand for factors production to monetary stimuli. In this case, an expansionary monetary policy causes excess demand and inflation rather than increasing aggregate supply. Consequently, monetary authorities find it optimal to focus on inflation stabilisation, which can be more effective in controlling than closing the output gap.

However, this is counter intuitive to the overall objective of monetary policy in developing economies, which is that the level of output is given the highest priority as indicated by the higher positive level at which inflation is stabilised compared to developed economies. The higher average rate of inflation stabilization level implies that money growth rate is higher and real interest rate is lower in developing countries compared to developed countries, to stimulate output growth rate.

Developing economies like that of Kenya require a higher rate of economic growth with the to improving incomes that influence social welfare. Besides, a higher output growth rate increases the speed with which the output gap is closed. In this regard, monetary authorities change policy interest rate with the intention of achieving a higher and steady rate of growth in national income. Hence, in table 1 column two, quadratic loss function is reestimated with national income growth rate instead, of actual national income gap to clarify the results in column 1 of table 1. The coefficient on income growth rate is negative and statistically insignificant as well as of almost the same size as that of actual income gap reported in column 1. Even though the results indicate that monetary authorities do not respond significantly to deviations in output as well as output growth rate from the output long trend and growth rate target, respectively, this may be due to misspecification of the model, as the model estimate does not take into account time variation in volatility. On the one hand, monetary authorities consider volatility in output and asset prices, because they affect commodity prices and returns on investment, which in turn influence output stability. This is one of the monetary policy objectives. On the other hand, the average household is risk averse. Risk, as measured by volatility, enters its welfare function explicitly as the variance. Thus, the social welfare function, which monetary authority maximises to optimise social welfare estimated, is misspecified if volatility is omitted in the regression equation.

Therefore, to overcome omitted variable bias in the social welfare parameter estimation, time varying volatility is included as an explanatory variable in the quadratic loss function in columns 3 to 5 of table 1. The parameter estimates in column 4 and 5 indicate that, monetary authorities respond to volatility by increasing policy rate, since the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalised Autoregressive Conditional Heteroscedasticity (GARCH) terms are positive and statistically significant. The volatility in the welfare function is positive and significant as well as the largest in the parameter estimates with output growth rate. This indicates that monetary authorities accord the greatest weight in responding to volatility in the social welfare function. A possible explanation of the response is that volatility, which is a measure of uncertainty and risk in the financial market as well as the entire economy, affects prices, consumption and investment decisions. Consequently, it affects welfare adversely especially for risk averse agents. As a result, monetary authority act strongly against volatility to mitigate its adverse effects.

With respect to output deviation from long trend and output growth rate deviation from the target growth, monetary authorities reduce the policy rate statistically significantly. The weight of its response is about 0.017 and 0.08 respectively. The weight monetary authority employs in responding to output deviation suggests an expansionary monetary policy. The response can be explained by the need to accelerate growth income, which is one of the main objectives of monetary policy in developing economies. Higher economic growth improves income per capita as well as disposable income of people in an economy which they use to spend on goods and service. Besides higher disposable income as a result of higher income growth rate, the income fluctuation smoothing monetary policy reduces fluctuations in households' income, which increases welfare.

Even after controlling for volatility, the exchange rate coefficient is positive and statistically significant. In column 5 of table 1, the Central Bank imposes a penalty of 0.21 per cent when exchange rate deviates from the desired path by 1 per cent. In a small open economy like that of Kenya, exchange rate movements affect aggregate demand first, by changing domestic prices relative to foreign prices, thereby switching demand between foreign and domestic produced goods. Secondly, exchange rate affect producer and consumer price indices, which monetary policy seeks to control. Hence, the response of monetary authorities to exchange rate changes. Finally, arbitrage in the international financial market increase sensitivity of exchange rate to interest rate differential between domestic and foreign markets. Furthermore, exchange rate movements affect returns on foreign denominated asset that influence portfolio investment and wealth of investors as well as consumption and welfare of agent. Therefore, exchange rate movement not only affect private agents, portfolio decision, but also expectations about future wealth. This is a possible explanation for the significant response of monetary policy against exchange rate deviations from the desired path. The result is consistent with Divino's (2009) analysis of optimal monetary policy in a small open economy.

The Central Bank responds to deviations in the domestic money market interest rates with a weight 0.356 per cent for a 1 per cent deviation from the long run level. The Central Bank intervenes in a credit money economy by changing the interest rate that it lends to commercial banks, which in turn influences market interest rate, exchange rate, inflation, yield on bonds and equity prices. In this regard, monetary authorities can achieve stability in asset prices and output by influencing market interest. In addition, money market interest rate is an intermediate target which is controlled by monetary authority with an aim to influence commodity and financial asset prices, stabilise prices and output, as well as enhancing output growth rate. Specifically, a positive deviation of inflation from its target requires an increase in the policy rate, which leads to increase in money market interest rate as well as a fall in bond prices. This not only reduces liquidity in the economy but also bond holders' wealth, given that bond prices fall when interest rates increase. As a result, consumption and aggregate demand falls. This drives actual inflation towards the target inflation rate. Hence, the significant weight monetary authority attaches to a deviation of money market interest rate from their long term level in the quadratic loss function. A deviation of inflation from its target by 1 per cent leads to an increase in the policy rate of about 0.028 per cent. The response of monetary authority is only significant when volatility is controlled for in estimating the quadratic loss function.

Financial asset prices have the least and statistically significant weights in the estimated monetary policy quadratic loss function in all the specifications. This indicates that the Central Bank does respond strongly to deviations of bond and equity prices. A possible explanation is that stock prices, even though have a significant effect on welfare and monetary stability, are driven by productivity, which monetary policy has little effect. However, monetary policy can respond to deviations of financial asset prices when they deviate from the discounted present value of dividends of shares or interest earning on bonds.

In as much as parameter estimates of the quadratic function provide relative weights the Central Bank attaches on output and prices in the social welfare function, the weights so estimated do not take into account optimisation decision of agents in the economy. The optimisation decision of agents in the financial and goods market as well as the government spending decisions affect actual and expected prices and output. Therefore, monetary authority can only be effective, if it the weights with which it responds to output and prices take into account optimisation decisions of agents in the economy. Furthermore, agents create expectations about monetary authority's response to deviations of output and prices from their socially desirable level to maximise social welfare (Kydland & Prescott, 1977; Di Bartolomeo & Giuli, 2011). Even though agents' expectations affect monetary policy response to deviations of an objective variable from its target, it has little control over such expectations. Thus, in what follows, the paper next attempts to find the optimal response to deviations of output and prices from the target path, taking into account explicitly the optimisation decisions of agents.

4.1 Optimal monetary policy with output and price volatility

This section utilises the household, firm and government optimisation behavior to analyse optimal monetary policy response to instability in output, inflation as well as in equity and bond prices. An optimal policy involves a choice of weight γ in equation 1 given optimal portfolio holding and income growth. The one period weight estimated from monetary policy optimisation problem indicates that monetary authority should assign a penalty of 0.0965, 0.0358, 0.0221, 0.0018, and 0.0223 on output, inflation, equity and bond prices instability respectively to maximise social welfare[7]. The results from the solution of optimisation problem using the numerical method suggest that monetary authority should respond to output deviating from long term trend with greatest weight compared to inflation, and asset prices. Indeed, a developing economy requires a higher rate of output growth as well as stability in income to improve welfare of the public. This is why monetary

[7] This corresponds to penalties of 0.0505, 0.0018, 0.0014, 0.0001 and 0.0182 for a unit deviation from the target rate for output, inflation, equity and bond prices respectively.

authority places greater emphasis on output target despite price stability objective being equally. This is important for developing economies in general and particularly in Kenya where supply side constraint in the real sector have a substantial effect on inflation and financial asset price instability.

The lower weight assigned to inflation in the social welfare is consistent with the view that stabilising inflation at very low levels is not optimal in a developing economy. This is because, firstly, complete inflation stabilisation requires higher variability in the market interest rate when monetary authority changes policy interest rates in response to shocks that affect commodity price level. Higher variability in market interest rates disrupts allocation of credit by lenders and impedes predictions about evolution of asset prices and returns on investment by investors, thereby reducing incentive to invest (Stulz, 1986). Secondly, a developing economy's monetary policy authority is eager to accelerate income growth to reduce poverty levels. However, accelerating income growth requires lower interest rate and a nominal exchange rate depreciation in a small open economy to increase exports. As a result, inflation deviates from zero due to lower domestic interest rate and nominal exchange rate pass-through effect to domestic prices (Divino, 2009; Monacelli, 2003). This results in the large difference in the weights monetary authority place on the output growth rate gap and inflation in the optimal solution, leading to the trade-off between output gap and inflation stabilisation.

Even though monetary authority has a higher propensity to increase income growth, it places substantial weight on stabilisation of bond prices compared to equity prices. This can be explained by the fact that treasury bonds traded on the NSE constitute a significant turnover of bond sales on the bonds market as well as in the entire stock market. Besides bonds trading being a significant portion of the capital market, bonds issued at the primary market change the level of liquidity and overall wealth of bond holders. This subsequently influence inflation. Therefore, substantial deviation of bond prices from their long term trend have a potential of inducing instability in the stock market and also in the entire financial sector, hence the higher weight bonds have relative to equity prices in the optimal solution.

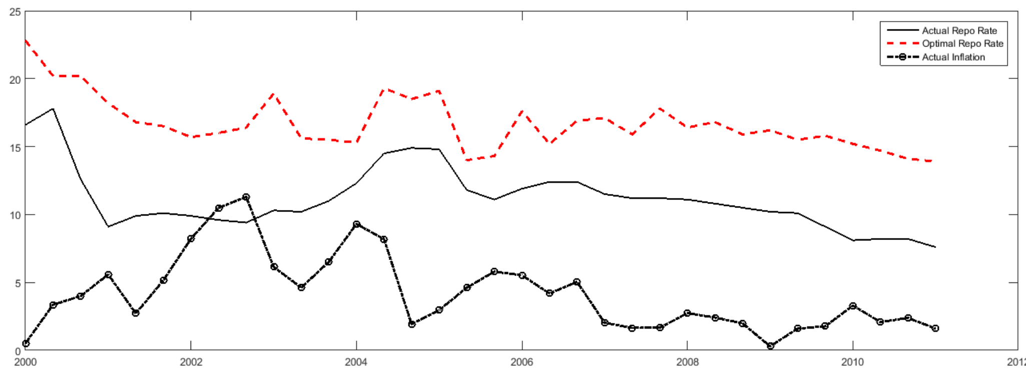
5.4.1.1. Monetary Policy under Discretion Rule

The monetary authority considers expectations of the private sector regarding inflation, interest rate and output when solving the optimisation problem for the society. In particular, monetary policy achieves inflation and output gap targets if inflation and output expectation private sector are properly anchored[8]. In this regard, monetary authority responds to current output, inflation, and asset price instability considering the current and future expectation of the private agents. Under discretion, the monetary authority optimises the welfare function period by period. Figure 1 presents the optimal level of policy interest rate and actual policy rate obtained from solving

[8] The expectations of the private sector regarding future monetary policy actions are rationalised that the current and future monetary policy decisions are consistent with current and future expected output gap and inflation rate.

a dynamic program of the monetary authority outline by equation 1 - 5. The optimal path for the monetary policy interest is solved using numerical methods by minimising the quadratic loss function each period with core inflation target restricted to between 3.75 per cent and 7.25 per cent. This is the monetary policy inflation target range set by the Central Bank of Kenya.

Figure 1: **Interest rate Path rule under Discretion**

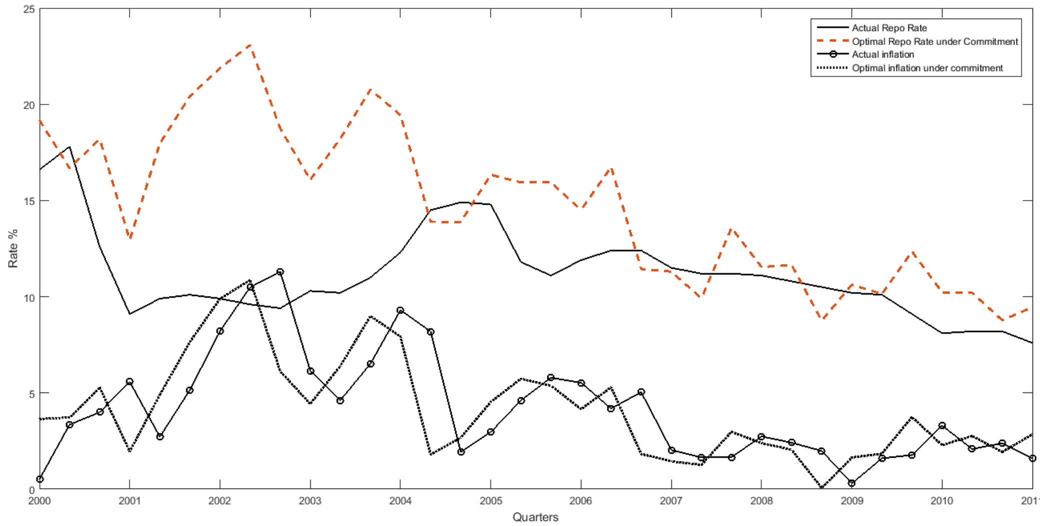


Even though discretion in this case results in a policy interest rate path that enables the Central Bank to achieve an inflation target, the resultant optimal output growth rate of 1 per cent is far much lower than actual average growth rate of 4.3 per cent and the target output growth rate of 5.8 per cent. This implies that the path for the optimal rate is higher than the actual policy rate, exerting a contractionary effect on the output growth that undermines achieving output growth rate objective. In as much as the path of monetary policy interest rate achieves inflation target, it is too high to be consistent with the higher income growth rate that is essential for welfare maximisation. The high interest rate reduces returns to investment that discourage meaningful investment in the economy. This leads to low output growth rate.

One of the solutions to inconsistent optimal interest rates with respect to final targets of monetary policy is commitment. Under commitment, monetary authority maximises the social welfare subject to agent's optimisation decision and credibly commits to the resultant optimal interest rate path, without deviating from the path. Figure 2 below plots the path resulting from the optimal solution of the social welfare function under commitment.

5.4.1.2. Monetary Policy Commitment Rule

Figure 2: **Path of interest rate rule under Commitment**



The optimal path of monetary policy interest rate enables optimal inflation to track actual inflation consistently overtime as shown in figure 2. The social loss value is 1198.152 with commitment to optimal rule, which is lower compared to welfare value 3183.620 under discretion. This implies that commitment monetary policy rule enables the society to attain a higher welfare compared to monetary policy rule under discretion. The results are consistent with the views of King (1997), Kydland & Prescott (1977) and Svensson (1997) that commitment and discretion attain only second and fourth best equilibrium respectively. The lower welfare under commitment can be attributed to monetary policy being not able to respond to unanticipated shocks, while remaining committed to optimal rule. One of the shocks that affect the optimal solution is the exchange rate movements especially in open small open economy like Kenya. Such unanticipated shocks induce significant changes in the output gap, asset and commodity price volatility but, monetary authority has no incentive to respond to them once an optimal path for interest rate rule has been determined, as in this case in which nominal exchange rate volatility has zero penalty in the optimal solution. The implication of zero weight on exchange rate in the optimal solution is that the monetary authority cannot respond to exchange rate directly without affecting optimal welfare value under commitment[9]. In addition, commitment to the optimal rule reduces the ability of monetary authority to take into account dynamic expectation of private agent, when consistently responding to output and asset price deviations, yet private agents' expectations influence monetary policy target variable. These findings are consistent with views of (Divino, 2009).

[9]However, reducing output gap and domestic inflation reduces demand for foreign currency which affect nominal and real exchange. In this way, monetary policy can at least influence exchange rate indirectly under commitment in a small open economy with strong exchange rate pass-through to domestic prices.

5 Conclusion

This paper focused on optimal monetary policy response to output and financial asset price instability. The regression results indicate that monetary policy should respond to interest rate instability more strongly compared to deviation of output growth rate from the set target. Monetary policy should also respond to distortions in the equity and bond prices with lesser weight compared to exchange rate. There is also evidence that deviation of inflation from the target can be restored by smoothening interest rate adjustment towards the target interest rate. The dynamic optimisation results suggest that the output and price objectives can be achieved optimally with a consistent and credible monetary policy. In addition, the results are consistent with the regression estimates of output and inflation rate weights.

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Appendix I
utility function
consumption C_t money $m_t$leisure n_t assets b_t.

$$V_t = E_t \sum_{t=0}^{\infty} \beta^t [\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} + \frac{M_t^{1-\eta}}{1-\eta}] \tag{6}$$

budget constraint

$$P_tC_t+E_t\left[\Psi_{t,t+1}B_{t+1}\right]+P_t\int_0^1 Q_t(i)Z_{t+1}\partial i+I_t+\frac{M_{t+1}}{(1+i_t)P_{t+1}} = W_tN_t+r_tK_t-P_tt_t+B_t+P_t\int_0^1[Q_t(i)+D_t(i)]Z_{t+1}\partial i+\frac{M_t}{P_t} \tag{7}$$

where$\Psi_{t,t+1} = \frac{1}{1+i_b}$is the nominal gross return on bonds. $I_t = (1-\delta)K_t + K_{t+1}$investment function.
first order conditions

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} - \frac{\lambda_t}{P_t} = 0$$

$$\frac{\partial L}{\partial N_t} = -N_t^{\gamma} + \lambda_t W_t = 0$$

$$\frac{\partial L}{\partial B_{t+1}} = -\lambda_t \Psi_{t,t+1} + \beta\lambda_{t+1} = 0$$

$$\frac{\partial L}{\partial Z_{t+1}} = -\lambda_t Q_t P_t + \beta\lambda_{t+1}[Q_{t+1} + D_{t+1}]P_{t+1} = 0$$

$$\frac{\partial L}{\partial K_{t+1}} = -\lambda_t + \beta\lambda_{t+1}[r_{t+1} + (1-\delta)] = 0$$

$$\frac{\partial L}{\partial M_t} = M_t^{\eta} + \frac{\lambda_t}{P_t} - \beta\frac{\lambda_{t+1}}{P_{t+1}} = 0$$

solving for equilibrium by eliminating λ_t and adjusting prices with nominal exchange rateε_t gives:

$$C_t = \left[\beta\left\{\frac{P_t}{P_{t+1}}\frac{\varepsilon_t}{\varepsilon_{t+1}}\left(1 + r_{t+1} - \delta\right)\right\}\right]^{-\frac{1}{\sigma}} C_{t+1} \tag{8}$$

$$N_t = \left[C_t^{-\sigma} W_t\right]^{\frac{1}{\gamma}}$$

$$\Psi_{t,t+1} = \beta\left[1 + r_{t+1} - \delta\right]$$

$$Q_t = \left[\frac{P_{t+1}}{P_t}\frac{\varepsilon_{t+1}}{\varepsilon_t}\left(Q_{t+1} + D_{t+1}\right)\right] / \left(1 + r_{t+1} - \delta\right)$$

$$M_t^{\eta} = -\frac{C_t^{-\sigma}}{\varepsilon P_{tt}} + \rho\frac{C_{t+1}^{-\sigma}}{(1+i_t)\varepsilon_{t+1}P_{t+1}}$$

$$\Psi_{t,t+1} = \rho\frac{P_t\varepsilon_t}{P_{t+1}\varepsilon_{t+1}}\frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}}$$

Firm

Technology is Cobb Douglas production function.

$$Y_t = N_t^{1-\alpha} e^{A_t}$$

$A_t = \rho A_{t-1} + \varrho_t \epsilon$ where $\varrho_t = \varpi\varrho_{t-1} + \nu_t$ is the variance of technology shock ϵ. Profit maximisation of firms is given by:

$$\Pi_t = Y_t - W_t N_t + \lambda_t\left(N_t^{1-\alpha}e^{A_t} - Y_t\right)$$

first order condition:

$$\frac{\partial \Pi_t}{\partial N_t} = -W_t + \lambda_t(1-\alpha)K_t^{\alpha}N_t^{-\alpha}e^{A_t} = 0$$

First order condition of profit maximization together with household utility optimisation determine equilibrium wage and real interest rate.

Price setting by the firm.

A fraction of firms Θ have market power, that enables then to adjust price in staggered manner over time while $(1-\Theta)$ take price as given (Calvo, 1983). Its is price setting behavior of firms that persistence of inflation and price rigidities in the economy. The Calvo price setting mechanism firm with market power is given as

$$\underset{P_t^*}{max} E_t \sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_t^* Y_{t+\tau|t} - MC_{t+\tau}Y_{t+\tau|t}\right)\right\}$$

$$Y_{t+\tau|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\Xi} C_{t+\tau}$$

$$\frac{\partial Y_{t+\tau|t}}{\partial P_t^*} = -E\left(\frac{P_t^*}{P_{t+k}}\right)^{-\Xi-1} C_{t+\tau}$$

$$= -\Xi\left(\frac{P_t^*}{P_{t+k}}\right)^{-\Xi-1} Y_{t+\tau}$$

$TR_{t+\pi} + MR_{t+\tau} - MC_{t+\tau}$

$$P_{t+\tau}Y_{t+\tau|t} + \frac{\partial Y_{t+\tau|t}}{\partial P_t^*} - MC_{t+\tau}\frac{\partial Y_{t+\tau|t}}{\partial P_t^*}$$

$$= P_{t+\tau}Y_{t+\tau|t} + P_t^*\left[-\Xi\left(\frac{P_{t+\tau}}{P_t^*}\right)Y_{t+\tau}\right] - MC_{t+\tau}\left[-\Xi\left(\frac{P_{t+\tau}}{P_t^*}\right)Y_{t+\tau}\right]$$

$$= P_{t+\tau}Y_{t+\tau|t}\left\{1 + P_t^*\left[-\Xi\left(\frac{1}{P_t^*}\right)\right] - MC_{t+\tau}\left[-\Xi\left(\frac{1}{P_t^*}\right)\right]\right\}$$

$$= P_{t+\tau}Y_{t+\tau|t}\left\{P_t^* + MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\right\}$$

$$E_t\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_{t+\tau}Y_{t+\tau|t}\left\{P_t^*+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\right\}\right)\right\}$$

then $E_t\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_{t+\tau}Y_{t+\tau|t}\left\{\frac{P_t^*}{P_{t+\tau}^*}+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\right\}\right)\right\}x\frac{P_{t+\tau}}{P_{t+\tau}}\frac{1}{P_{t+\tau}}$
gives

$$E_t\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\Gamma_{t,t+\tau}\left(P_{t+\tau}Y_{t+\tau|t}\left\{\pi^*+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\pi_{t+\tau}\right\}\right)\right\}$$

note that $\Gamma_{t,t+\tau}=\beta\left[\left(\frac{C_t}{C_{t+\tau}}\right)^{-\sigma}\frac{P_t}{P_{t+\tau}}\right]$
hence

$$E_t\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\beta\left[\left(\frac{C_t}{C_{t+\tau}}\right)^{-\sigma}\frac{P_t}{P_{t+\tau}}\right]\left(P_{t+\tau}Y_{t+\tau|t}\left\{\pi^*+MC_{t+\tau}\left(\frac{\Xi}{1-\Xi}\right)\pi_{t+\tau}\right\}\right)\right\}$$

$$E_t\sum_{\tau=0}^{\infty}\Theta^{\tau}\left\{\beta\left[y_t-\sigma(c_{t+\tau}-c_t)+p_t-p_{t+\tau}+y_{t+\tau}+\pi_t^*-(y_t-\sigma(c_{t+\tau}-c_t)+y_{t+\tau}+mc_{t+k})\right]\right\}$$

which can be summarized as

$$E_t\sum_{\tau=0}^{\infty}(\Theta\beta)^{\tau}\left[\pi_t^*+p_t\right]=E_t\sum_{\tau=0}^{\infty}(\Theta\beta)^{\tau}\left[mc_{t+\tau}+p_{t+\tau}\right]$$

solving infinite summation and using $\pi_t=(1-\Theta)\pi_t^*$

$$\pi_t=\frac{(1-\Theta)(1-\Theta\beta)}{(\Theta)}mc_t+\beta E_t(\pi_{t+1})$$

is the Philips curve.
where $mc_t=\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{W_t^{1-\alpha}r_{k,t}^{\alpha}}{A_t}$ real marginal cost. Equation states that firms maximize their discounted revenue by choosing price P_t^*.
log linearing and approximating using first order Taylor expansion.
Openness

$$P_t=\left[(1-\Phi)\left(P_{H,t}\right)^{1-\emptyset-1}+\Phi\left(P_{F,t}\right)^{1-\emptyset}\right]^{\frac{1}{1-\emptyset}} \tag{9}$$

where$\Phi$is the degree of openness, $\emptyset$ is the substitution parameter which measures the ease of substitution between domestic and foreign goods .
log linearising price

$$P_t=\left[(1-\Phi)\left(P_{H,t}\right)+\Phi\left(P_{F,t}\right)\right]$$

Terms of trade
Bilateral terms of trade between the home country i and trading partner country j is $S_{i,t}=\frac{P_{F,t}}{P_{H,t}}$ It is the relative price index for each country. In log form $s_t=P_{F,t}-P_{H,t}$. $P_{F,t}$ and $P_{H,t}$ are the foriegn and the domestic price indices respectively.

$$P_t=\left[(1-\Phi)\left(P_{H,t}\right)+\Phi\left(P_{F,t}\right)\right]=P_{H,t}+\Phi s_t \tag{10}$$

low of one price $P_{i,jt}=\varepsilon_{it}P_{jt}^i$ is the purchasing power parity, stating that foreign price should be equal to domestic price in domestic currency.$\varepsilon_t=\psi\varepsilon_{t-1}+\omega_t\hbar_t$ is the nominal exchange rate which follows an Auto regressive process of order 1 (AR1). In managed floating regimes exchange rate movements approximate AR 1 process (Funke et al., 2011) . $\omega_t=\rho_{\omega,t}\omega_{t-1}+\upsilon_t$ stochastic volatility process in the nominal exchange rate. $\hbar_t\sim N(0,1)$ $\nu_t\sim N(0,1)$.
foreign price $P_{F,t}=\int_0^1\left(e_{i,t}+P_{i,t}^i\right)\partial i=e_t+P_t^*$

$$s_t=e_t+P_t^*-P_{H,t}$$

Real exchange rate$\mathbb{R}$

$$\mathbb{R}=\varepsilon_{i,t}\frac{P_t^i}{P_t}$$

$$=\int_0^1\left(e_{i,t}+P_{i,t}^i-P_t\right)\partial i$$

$$=e_t+P_t^*-P_t$$

$$=s_t+P_{H,t}-P_t$$

$$= (1 - \Phi)s_t$$

world in incomey_t^* is an autoregressive process of order 1.

$$y_t^* = \psi y_{t-1}^* + \zeta_t$$

Appendix II

$$V_t = E_t \sum_{t=0}^{\infty} \beta^t [\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} + \frac{M_t^{1-\eta}}{1-\eta}]$$

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{\overline{C_t}^{1-\sigma}}{1-\sigma} + \overline{C_t}^{1-\sigma}\left[\overline{C_t} + \frac{(1-\sigma)\overline{C_t}^2}{2}\right] + \Upsilon^3$$

$$\frac{N_t^{1+\gamma}}{1+\gamma} = \frac{\overline{N_t}^{1+\gamma}}{1+\gamma} + \overline{N_t}^{1+\gamma}\left[\overline{N_t} + \frac{(1+\gamma)\overline{N_t}^2}{2}\right] + \Upsilon^3$$

$$\frac{M_t^{1-\eta}}{1-\eta} = \frac{\overline{M_t}^{1-\eta}}{1-\eta} + \overline{M_t}^{1-\eta}\left[\overline{M_t} + \frac{(1-\eta)\overline{M_t}^2}{2}\right] + \Upsilon^3$$

Second order Taylor approximation for output at zero state state.

$$Y_t = \overline{Y} + \overline{Y}Y_t + \frac{1}{2}\overline{Y}Y_t^2 + \Upsilon^3$$

Consumption $\overline{C}$is given by $\overline{C} = \overline{Y} - \overline{S}$

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{\overline{C_t}^{1-\sigma}}{1-\sigma} + \overline{C_t}^{1-\sigma}\left[\overline{Y_t} - \overline{S_t} + \frac{(1-\sigma)\left(\overline{Y_t} - \overline{S_t}\right)^2}{2}\right] + \Upsilon^3$$

$$N_t = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\xi} \partial i$$

let $d_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\xi} \partial i$ The $n_t = y_t + d_t - a_t$ in logarithim form. let also $P_{H,t}^{\acute{}}(i) = P_{H,t}(i) - P_{H,t}$ by taking the logarithim. A second order Talyor series approximation after taking logarithim. Hence:

$$\begin{aligned}
\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\xi} &= exp\left[(1-\xi)P_{H,t}^{\acute{}}(i)\right] \\
&= 1 + (1-\xi)P_{H,t}^{\acute{}}(i) + \frac{1}{2}(1-\xi)^2\{P_{H,t}^{\acute{}}(i)\}^2 + \Upsilon^3 \\
E_i\{P_{H,t}^{\acute{}}(i)\} &= \frac{1}{2}(\xi - 1)E_i\{P_{H,t}^{\acute{}}(i)\}^2
\end{aligned}$$

$$\begin{aligned}
\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\xi} \partial i &= 1 - \xi E_i\{P_{H,t}^{\acute{}}(i)\} + \frac{1}{2}\xi^2 E_i\{P_{H,t}^{\acute{}}(i)\}^2 + \Upsilon^3 \\
&= 1 + \frac{\xi}{2}E_i\{P_{H,t}^{\acute{}}(i)\}^2 + \Upsilon^3 \\
&= 1 + \frac{\xi}{2}Var_i\{P_{H,t}^{\acute{}}(i)\} + \Upsilon^3
\end{aligned}$$

and therefore $d_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\xi} \partial i = \frac{\xi}{2}Var_i\{P_{H,t}^{\acute{}}(i)\} + \Upsilon^3$

$$\begin{aligned}
\frac{N_t^{1+\gamma}}{1+\gamma} &= \overline{N_t}^{1+\gamma}\left[\overline{N_t} + \frac{(1+\gamma)\overline{N_t}^2}{2}\right] + \Upsilon^3 \\
&= \overline{N_t}^{1+\gamma}\left[y_t + d_t - a_t + \frac{(1+\gamma)(y_t + d_t - a_t)^2}{2}\right] + \Upsilon^3
\end{aligned}$$

$\overline{N_t}^{1+\gamma} = (1-\alpha)$ That is maginal utility of labour equal to the share of labour in the national output. An optimal subsidy also enables labour suppliers to have this share.

Hence

$$\frac{N_t^{1+\gamma}}{1+\gamma} = (1-\alpha)\left[d_t + \frac{(1+\gamma)(y_t)^2}{2}\right] + tip + \Upsilon^3$$

tip consists of $a_t, d_t a_t, y_t d_t a_t$and y_t which cannot be influenced by monetary policy.

Second order Talyor approximation for equity and bond price.

Financial asset consits of money bonds and equity.

$$\Psi = M + B_t \int_0^1 \left(\frac{F_{H,t}(i)}{F_{H,t}}\right)^{-\epsilon} \partial i + Q_t \int_0^1 \left(\frac{Z_{K,t}(i)}{Z_{K,t}}\right)^{-\omega} \partial i$$

Following a similar procedure as inflation, equity and bond price approximations are,

$$\widetilde{B_t} = \int_0^1 \left(\frac{F_{H,t}(i)}{F_{H,t}}\right)^{-\epsilon} \partial i = 1 + \frac{\epsilon}{2} Var_i\{\acute{F_{H,t}}(i)\} + \Upsilon^3$$

$$\widetilde{Q_t} = \int_0^1 \left(\frac{Z_{K,t}(i)}{Z_{K,t}}\right)^{-\omega} \partial i = 1 + \frac{\omega}{2} Var_i\{\acute{Z_{K,t}}(i)\} + \Upsilon^3$$

$$\begin{aligned}
\frac{M_t^{1-\eta}}{1-\eta} &= \overline{M_t}^{1-\eta}\left[(\overline{B_t} + \widetilde{B_t} + \overline{Q_t} + \widetilde{Q_t}) + \frac{(1-\eta)(\overline{B_t} + \widetilde{B_t} + \overline{Q_t} + \widetilde{Q_t})^2}{2}\right] + \Upsilon^3 \\
&= \alpha\left[(\overline{B_t} + \widetilde{B_t} + \overline{Q_t} + \widetilde{Q_t}) + \frac{(1-\eta)(\overline{B_t} + \widetilde{B_t} + \overline{Q_t} + \widetilde{Q_t})^2}{2}\right] + \Upsilon^3 \\
&= \alpha\left[(\widetilde{B_t} + \widetilde{Q_t}) + \frac{(1-\eta)(\widetilde{B_t} + \widetilde{Q_t})^2}{2}\right] + tip + \Upsilon^3
\end{aligned}$$

substituting all the terms in the utility function

$$\begin{aligned}
V_t &= E_t \sum_{t=0}^{\infty} \beta^t [\alpha\left[\frac{y_t^2}{2} + \frac{(1-\sigma)}{2} y_t^2\right] - (1-\alpha)\left[d_t + \frac{(1+\gamma)(y_t)^2}{2}\right] + \alpha\left[(\widetilde{B_t} + \widetilde{Q_t}) + \frac{(1-\eta)(\widetilde{B_t} + \widetilde{Q_t})^2}{2}\right] + tip + \Upsilon^3] \\
&= E_t \sum_{t=0}^{\infty} \beta^t [\alpha \frac{(1-\sigma)}{2} y_t^2 - \frac{(1-\alpha)\pi_{H,t}^2}{2\Gamma} - \frac{(1-\alpha)(1+\gamma)(y_t)^2}{2} + \frac{\alpha\omega}{2}\widetilde{Q_t}^2 + \alpha\frac{\epsilon}{2}\widetilde{B_t}^2] + tip + \Upsilon^3 \\
&= E_t \sum_{t=0}^{\infty} \beta^t [\frac{(1-\alpha)\pi_{H,t}^2}{2\Gamma} + \frac{\alpha(1-\sigma) - (1-\alpha)(1+\gamma)(y_t)^2}{2} + \frac{\alpha\omega}{2}\widetilde{Q_t}^2 + \alpha\frac{\epsilon}{2}\widetilde{B_t}^2] + tip + \Upsilon^3
\end{aligned}$$

where $\Gamma = \frac{(1-\Theta)(1-\Theta\beta)}{(\Theta)}$ from the Keynesian Philips curve.

Then the continuous counterpart is

$$L = E_t \int_{t=0}^{\infty} e^{-\delta t}[\frac{(1-\alpha)\pi_{H,t}^2}{2\Gamma} + \frac{\alpha(1-\sigma) - (1-\alpha)(1+\gamma)(y_t)^2}{2} + \frac{\alpha\omega}{2}\widetilde{Q_t}^2 + \alpha\frac{\epsilon}{2}\widetilde{B_t}^2] + tip + \Upsilon^3$$

$$\underset{Maximise}{L} = \int_{t=0}^{\infty} e^{-\delta t}\frac{1}{2}\left(\gamma_\pi \pi_{H,t}^2 + \gamma_y (y_t)^2 + \gamma_Q \widetilde{Q_t}^2 + \gamma_B \widetilde{B_t}^2\right) \tag{11}$$

Where$\gamma_\pi = \frac{(1-\alpha)}{\Gamma}$, $\gamma_y = \alpha(1-\sigma) - (1-\alpha)(1+\gamma)$,$\gamma_Q = \alpha\omega$, $\gamma_B = \alpha\epsilon$

given

$$\mathring{y} = \beta x + b((1+i) - 1.2) + e \tag{12}$$

$$\mathring{\pi} = ((1+i) - 1.2) + Cx \tag{13}$$

$$\mathring{Q} = \frac{(1.38\pi\varepsilon_t)}{1+i} \tag{14}$$

$$\mathring{B} = \frac{1.02}{(1+i)} \tag{15}$$

$$\mathring{x} = \frac{1.02(1+i) - B}{(1+i)} + ty \tag{16}$$

$$\mathring{\varepsilon}_t = -i \tag{17}$$

$$i > 0$$

δ discount factor, t time.

Equation 11 is the state equation, and L is the variable to be maximised.γ_π, γ_y, γ_Q, γ_B are contributions of inflation, income, equity price and debt instability to social loss. They are also weights of respective variable in the social loss function. Equation 2-7 are decision equation which influence the state equation 1. The decision variable is i interest rate. That is i is changed to ensure that L is maximised subject to equation 2-7. B is the debt level, y income, π inflation, x expenditure, e output shock, β sensitivity of change in income to spending, b sensitivity of output to inflation stability, Q equity price, t tax rate. r exchange rare. The dots above a variable indicate the variable is a first derivative with respect to time.

Table 2: Endogeneity Test

Null Hypothesis:	Obs	F-Statistic	Prob.
Output does not Granger Cause Repo rate	60	0.28646	0.8865
Repo rate does not Granger Cause Output		0.15067	0.9625
Exchange rate does not Granger Cause Repo rate	60	56.8864	2.00E-30
Repo rate does not Granger Cause Exchange rate		12.572	5.00E-09
Bond prices does not Granger Cause Repo rate	60	1.38807	0.2401
Repo rate does not Granger Cause Bond prices		1.11976	0.3489
Inflation does not Granger Cause Repo rate	60	0.71382	0.5835
Repo rate does not Granger Cause Inflation		2.77918	0.0285
Bond prices does not Granger Cause Repo rate	60	3.96336	0.0042
Repo rate does not Granger Cause Bond prices		1.4228	0.2284
Output growth rate does not Granger Cause Repo rate	60	0.32243	0.8627
Repo rate does not Granger Cause Output growth rate		0.45222	0.7707
Exchange rate does not Granger Cause Output	60	0.58039	0.6772
Output does not Granger Cause Exchange rate		0.63894	0.6354
Bond prices does not Granger Cause Output	60	1.52413	0.1973
Output does not Granger Cause Bond prices		0.16392	0.9564
Inflation does not Granger Cause Output	60	0.07116	0.9907
Output does not Granger Cause Inflation		1.19109	0.3165
Bond prices does not Granger Cause Output	60	0.31112	0.8703
Output does not Granger Cause Bond prices		0.21154	0.9318
Output growth rate does not Granger Cause Output	60	129.661	8.00E-51
Output does not Granger Cause Output growth rate		60.5361	8.00E-32
Bond prices does not Granger Cause Exchange rate	60	1.08836	0.3639
Exchange rate does not Granger Cause Bond prices		1.20501	0.3105
Inflation does not Granger Cause Exchange rate	60	0.12036	0.9751
Exchange rate does not Granger Cause Inflation		3.48684	0.0091
Bond prices does not Granger Cause Exchange rate	60	0.23693	0.9172
Exchange rate does not Granger Cause Bond prices		1.85172	0.1212
Output growth rate does not Granger Cause Exchange rate	60	0.15353	0.9612
Exchange rate does not Granger Cause Output growth rate		0.64703	0.6297
Inflation does not Granger Cause Equity prices	60	0.13883	0.9677
Equity prices does not Granger Cause Inflation		0.58226	0.6759
Bond prices does not Granger Cause Equity prices	60	1.79712	0.1316
Equity prices does not Granger Cause Bond prices		0.32664	0.8598
Output growth rate does not Granger Cause Equity prices	60	0.31541	0.8674
Equity prices does not Granger Cause Output growth rate		0.96377	0.4289
Bond prices does not Granger Cause Inflation	60	12.6493	5.00E-09
Inflation does not Granger Cause Bond prices		0.29382	0.8817
Output growth rate does not Granger Cause Inflation	60	0.61856	0.6499
Inflation does not Granger Cause Output growth rate		0.1651	0.9558
Output growth rate does not Granger Cause Bond prices	60	0.28705	0.8861
Bond prices does not Granger Cause Output growth rate		0.20375	0.936

Notes: In this regression the dependent variable is the deviation of repo rate from its trend. Output gap is the deviation of actual output from the potential output, which is the trend. Output growth rate is squared deviation from target growth rate. (Output is equal to income from national income account) inflation is the squared deviation from the average target inflation rate of 5 per cent. Equity, bond prices and interest are squared deviation from their long term trends. With respect to bond prices the long term trend is consistent with the yield curve.